FULLY WORKED SOLUTIONS

Context 1: Physics on the road

Chapter 3: Starting, stopping and moving

Chapter questions

1. $d = 10 \text{ cm} + 20 \text{ cm} = 30 \text{ cm}$

2. (a) $R = \sqrt{10^2 + 8^2} = \sqrt{100 + 64} = \sqrt{164} = 12.8 \text{ km}$

$\tan\theta = \frac{8}{10} = 0.8$

$\theta = \tan^{-1}(0.8) = 38.7^\circ \text{ south of west} = 218.7^\circ$

(b) $d = 10 \text{ km} + 8 \text{ km} = 18 \text{ km}$

3. 400 m west

4. (a) $d = 14 \text{ km} + 17 \text{ km} = 31 \text{ km}$

speed $= \frac{d}{t} = \frac{31 \text{ km}}{2 \text{ h}} = 15.5 \text{ km h}^{-1}$

(b) $s = \sqrt{14^2 + 17^2} = \sqrt{485} = 22.0 \text{ km}$

$\tan\theta = \frac{17}{14} = 1.2$

$\theta = \tan^{-1}(1.2) = 50.2^\circ$ north of east $= 50.2^\circ$

$\boldsymbol{v} = \frac{s}{t} = \frac{22 \text{ km at } 50.2^\circ}{2 \text{ h}} = 11.0 \text{ km h}^{-1} \text{ at } 50.2^\circ$

5. (a) speed $= \frac{30 \text{ cm}}{1.5 \text{ s}} = 20 \text{ cm s}^{-1}$

(b) $\boldsymbol{v} = \frac{30 \text{ cm east}}{1.5 \text{ s}} = 20 \text{ cm s}^{-1} \text{ east}$

6. (a) distance = 1 km + 6 km = 7 km

(b) time = 1 min + 5 min + 30 min = 36 min = 0.6 h

average speed $= \frac{d}{t} = \frac{7 \text{ km}}{0.6 \text{ h}} = 11.7 \text{ km h}^{-1}$

(c) At the beginning of the journey, the end of her journey and while she was in the shop, the car's instantaneous speed was equal to zero.

7. $u = 15 \text{ m s}^{-1}$, $v = 24 \text{ m s}^{-1}$, $t = 2.7 \text{ s}$

$a = \frac{v - u}{t} = \frac{24 - 15}{2.7} = 3.3 \text{ m s}^{-2} \text{ north}$

8. $u = 22 \text{ m s}^{-1}$, $a = -4 \text{ m s}^{-2}$, $v = 0$

Rearranging the equation $a = \frac{v - u}{t}$ we get $t = \frac{v - u}{a}$.

$t = \frac{0 - 22}{-4} = 5.5\ s$

9. $u = 0$, $a = 5\ \text{m s}^{-2}$ east, $t = 3.9$ s

Rearranging the equation $a = \frac{v - u}{t}$ we get $v = u + at$.

$v = 0 + (5)(3.9) = 19.5\ \text{m s}^{-1}$ east

10. $u = 10\ \text{m s}^{-1}$, $v = 30\ \text{m s}^{-1}$, $t = 10$ s

$s = ½t(v + u)$

$s = ½ \times 10 \times (30 + 10) = 200$ m

11. $u = 20\ \text{m s}^{-1}$, $s = 240$ m, $v = 3\ \text{m s}^{-1}$

Rearranging the equation $v^2 = u^2 + 2as$, we get $a = \frac{v^2 - u^2}{2s}$.

$a = \frac{3^2 - 20^2}{2(240)} = \frac{-391}{480} = -0.81\ \text{m s}^{-2}$

12. (a) $u = 0$, $s = 1$ m, $v = 4\ \text{m s}^{-1}$

Rearranging the equation $v^2 = u^2 + 2as$, we get $a = \frac{v^2 - u^2}{2s}$.

$a = \frac{4^2 - 0^2}{2(1)} = \frac{16}{2} = 8\ \text{m s}^{-2}$

(b) $u = 0$, $s = 1$ m, $v = 4\ \text{m s}^{-1}$, $a = 8\ \text{m s}^{-2}$

Rearranging the equation $v = u + at$, we get $t = \frac{v - u}{a}$

$t = \frac{4 - 0}{8} = 0.5\ \text{s}$

Review questions

4. (a) $d = 200\ \text{m} + 400\ \text{m} + 300\ \text{m} = 900$ m

(b) Overall, the car travels 500 m East and 400 m North.

$s = \sqrt{500^2 + 400^2} = 640\ \text{m}$

$\tan\theta = \frac{400}{500} = 0.8$

$\theta = \tan^{-1}(0.8) = 39°$ north of east $= 39°$

Therefore, $s = 640$ m at 39°

5. Consider the 3 sections as 3 vectors: 4 km at 90°, 6 km at 120° and 2 km at 270°.

We can now add these vectors using horizontal (east) components and vertical (north) components of the final displacement:

s_{east} = 4 cos 90° + 6 cos 120° + 2 cos 270°

= 0 + (–3) + 0

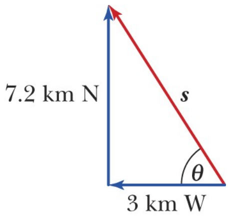
= –3 km

= 3 km west

s_{north} = 4 sin 90° + 6 sin 120° + 2 sin 270°

= 4 + 5.2 + (–2)

= 7.2 km north



$s = \sqrt{3^2 + 7.2^2} = \sqrt{60.84} = 7.8\ \text{km}$

$\tan\theta = \dfrac{7.2}{3} = 2.4$

$\theta = \tan^{-1}(2.4)$ = 67° N of W (or 113°)

s = 7.8 km at 113°

6. (a) speed = $\dfrac{d}{t} = \dfrac{100\ \text{km}}{1.5\ \text{h}}$

= 66.7 km h^{-1}

= 18.5 m s^{-1}

(b) speed = $\dfrac{d}{t} = \dfrac{1\ \text{km}}{2\ \text{min}} = \dfrac{1000\ \text{m}}{120\ \text{s}}$

= 8.3 m s^{-1}

(c) speed = $\dfrac{d}{t} = \dfrac{100\ \text{m}}{11.5\ \text{s}}$

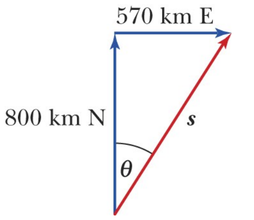
= 8.7 m s^{-1}

(d) speed = $\dfrac{d}{t} = \dfrac{200\ \text{m}}{4\ \text{min}} = \dfrac{200\ \text{m}}{240\ \text{s}}$

= 0.83 m s^{-1}

7. speed = $\dfrac{d}{t} = \dfrac{(800\ \text{km} + 570\ \text{km})}{2\ \text{h}}$

$= 685 \text{ km h}^{-1}$



$s = \sqrt{800^2 + 570^2} = 982.3 \text{ km}$

$\tan\theta = \frac{570}{800} = 0.71$

$\theta = \tan^{-1}(0.71) = 35.5°$ E of N (or 54.5°)

$v = \frac{982.3 \text{ km}}{2 \text{ h}} = 491 \text{ km h}^{-1}$ at $54.5°$

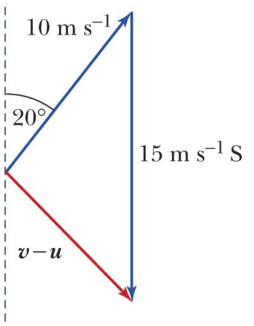
8. Known: $u = 20 \text{ m s}^{-1}$, $v = 28 \text{ m s}^{-1}$, t = 3.0 s; to find: a

$a = \frac{v - u}{t} = \frac{28 - 20}{3.0} = 2.7 \text{ m s}^{-2}$

9. Known: $a = –3 \text{ m s}^{-2}$, $t = 5.0$ s, $u = 80 \text{ km h}^{-1} = 22.2 \text{ m s}^{-1}$; to find: v

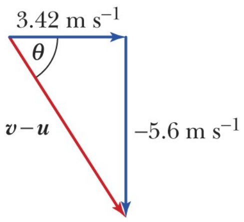
$v = u + at = 22.2 + (–3)(5.0) = 22.2 – 15 = 7.2 \text{ m s}^{-1}$

10. $a = \frac{\Delta v}{t} = \frac{v + (-u)}{t}$



$(v – u)_x = 10 \cos 70° + 15 \cos 270° = 3.42 + 0 = 3.42 \text{ m s}^{-1}$

$(v – u)_y = 10 \sin 70° + 15 \sin 270° = 9.4 + (-15) = - 5.6 \text{ m s}^{-1}$



$v – u = \sqrt{3.42^2 + 5.6^2} = 6.6 \text{ m s}^{-1}$

$\tan\theta = \frac{5.6}{3.4} = 1.6$

$\theta = \tan^{-1}(1.6) = 58°$ south of east (or 302°)

$$a = \frac{6.6 \ \text{m s}^{-1}}{2.3 \ \text{s}} \text{ at } 302° = 2.9 \ \text{m/s}^{-2} \text{ at } 302°$$

11. (a) Known: $u = 5$ m s^{-1}, $a = 1.6$ m s^{-2}, $t = 4.3$ s; to find: s

$$s = ut + \frac{1}{2}at^2 = (5 \times 4.3) + \frac{1}{2}(1.6 \times 4.3^2) = 21.5 + 14.8 = 36.3 \text{ m}$$

(b) Known: $u = 5$ m s^{-1}; $a = 1.6$ m s^{-2}; $t = 4.3$ s; $s = 36.3$ m; to find: v

$v = u + at = 5 + (1.6 \times 4.3) = 12$ m s^{-1}

(c) Known: $u = 5$ m s^{-1}, $a = 1.6$ m s^{-2}, $t = 3$ s; to find: s

$$s = ut + \frac{1}{2}at^2 = (5 \times 3) + \frac{1}{2}(1.6 \times 3^2) = 15 + 7.2 = 22.2 \text{ m}$$

(d) To find the distance travelled in the fourth second, s_4, we find the difference between the total displacement at the end of the fourth second and subtract the total displacement at the end of the third second.

Known: $u = 5$ m s^{-1}; $a = 1.6$ m s^{-2}; $t = 4$ s; to find: s

$$s = ut + \frac{1}{2}at^2 = (5 \times 4) + \frac{1}{2}(1.6 \times 4^2) = 20 + 12.8 = 32.8 \text{ m}$$

$s_4 = 32.8 - 22.2 = 10.6$ m

12. Known: $t = 8$ s, $u = 0$, $a = 2.2$ m s^{-2}; to find: v

$v = u + at = 0 + 2.2 \times 8 = 17.6$ m s^{-1}

13. Known: $u = 0$, $s = 200$ m, $t = 10$ s; to find: a

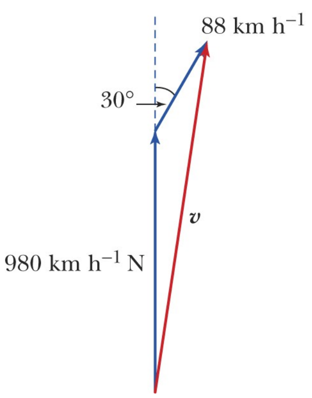
$$s = ut + \frac{1}{2}at^2$$

$$200 = (0 \times 10) + \frac{1}{2}(a \times 10^2)$$

$$200 = 50a$$

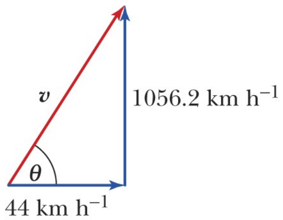
$a = 4$ m s^{-2}

14.



$v_x = 980 \cos 90° + 88 \cos 60° = 0 + 44 = 44$ km h^{-1}

$v_y = 980 \sin 90° + 88 \sin 60° = 980 + 76.2 = 1056.2$ km h^{-1}



$v = \sqrt{44^2 + 1056.2^2} = 1057.1$ km h^{-1}

$\tan \theta = \frac{1056.2}{44} = 24$

$\theta = \tan^{-1} 24 = 87.6°$ E of N (or 87.6°)

$v = 1057.1$ km h^{-1} at 87.6°

15. speed $= \frac{d}{t} = \frac{1609 \text{ m}}{239.4 \text{ s}} = 6.72 \text{ m s}^{-1}$

16. (a) $20 \div 3.6 = 5.6$ m s^{-1}

(b) $100 \div 3.6 = 27.8$ m s^{-1}

(c) $15 \times 3.6 = 54$ km h^{-1}

(d) $3\,000 \times 3.6 = 10\,800$ km h^{-1}

18. (a) At $t = 0$, $t = 10$ to 15 min, $t = 35$ to 40 min, $t = 50$ to 60 min

(b) Picking up or dropping off passengers; stopping for red lights

(c) $v_{inst} = \frac{\Delta s}{\Delta t} = \frac{25 - 10}{35 - 15} = 0.75 \text{ km min}^{-1} = 45 \text{ km h}^{-1}$

(d) $v_{av} = \frac{s}{t} = \frac{13 \text{ km}}{20 \text{ min}} = 0.65 \text{ km min}^{-1} = 39 \text{ km h}^{-1}$

19. Known: $u = 0$, $a = 2.4$ m s^{-2}, $t = 12$ s; to find: s

$s = ut + \frac{1}{2}at^2 = (0 \times 12) + \frac{1}{2}(2.4 \times 12^2) = 173$ m

20. (a) Tania (steepest gradient)

(b) Tania won.

(c) Tania overtook Jenny.

(d) Jenny had a 40 m head start over Tania. This could be considered fair if Tania was being handicapped because she was a better runner than Jenny normally, or unfair if they were both pretty much of the same standard normally.

21 (a) 60 km h^{-1}

(b) Displacement = area between line of motion and t-axis

At the 8 s mark, car B is moving at 54 km h^{-1} or 15 m s^{-1}

$s = \frac{1}{2}(8 \times 15) = 60$ m

(c) At 4 s from rest, car B is travelling at 27 km h^{-1} or 7.5 m s^{-1}.

$a = \frac{\Delta v}{\Delta t} = \frac{7.5 \text{ m s}^{-1}}{4 \text{ s}} = 1.9 \text{ m s}^{-2}$

(d) Car B will catch up to car A at the point in time when the area under their graphs is the same — that is, when they are at the same place at the same time. This will occur at $t = 15$ s.

22. Known: $u = 10$ m s^{-1}, $v = 22$ m s^{-1}, $s = 300$ m; to find: a

$v^2 = u^2 + 2as$

$22^2 = 10^2 + 2a \times 300$

$484 = 100 + 600a$

$384 = 600a$

$a = 0.64$ m s^{-2}

23. In order to win the race, Brett will have to run the 400 m in a shorter overall time than Tony.

Tony's time

Known: $u = 0$, $a = 1$ m s^{-2}, $t = 3.2$ s, $s_T = 400$ m; to find: s, v

$s = ut + \frac{1}{2}at^2 = (0 \times 3.2) + \frac{1}{2}(1 \times 3.2^2) = 5.1$ m

$v = u + at = 0 + (1 \times 3.2) = 3.2$ m s^{-1}

After the initial 3.2 s acceleration, Tony will run at 3.2 m s^{-1} over the next 394.9 m. The time this will take is:

$t = \frac{d}{v} = \frac{394.9 \text{ m}}{3.2 \text{ m s}^{-1}} = 123.4 \text{ s}$

Therefore, Tony's total time for the race is 3.2s + 123.4 s = 126.6 s.

In order to beat Tony, Brett will need to have a time less than 126.6 s.

Brett's time

Brett spends 4 s at rest, so he has to run 400 m in less than 122.6 s.

Known: $u = 0$, $a = 1.8$ m s^{-2}, $t = 2.8$ s; to find: s, v

$s = ut + \frac{1}{2}at^2 = (0 \times 2.8) + \frac{1}{2}(1.8 \times 2.8^2) = 7.1$ m

$v = u + at = 0 + (1.8 \times 2.8) = 5.0$ m s^{-1}

After his initial 2.8 s acceleration, Brett will run the remaining 392.9 m at 5.0 m s^{-1}.

$$t = \frac{d}{v} = \frac{392.9\ \text{m}}{5.0\ \text{m s}^{-1}} = 78.6\ \text{s}$$

Therefore, Brett's time for the 400 m is 2.8 s + 78.6 s = 81.4 s. Brett wins the race easily.

24. The dog will catch up to the car when they are at the same place at the same time, i.e at a time *t* when *s* is the same for both the car and the dog. As the dog and the car have been in motion for different periods of time, we will define t_d as the time that the dog has been in motion when it catches the car, and t_c as the time that the car has been in motion. Note that, as the dog started moving 1.2 s after the car went by, $t_c = t_d + 1.2$

Car

Known: $v_{av} = 15$ m s^{-1}; to find: s, t_c

As $v_{av} = \frac{s}{t}$, then $s = v_{av}t$

$s = 15t_c$

Dog

Known: $u = 0$, $a = 1.7$ m s^{-2}; to find: s, t_d

$s = ut + \frac{1}{2}at^2$

$= (0 \times t_d) + \frac{1}{2} \times 1.7 \times t_d^2$

$s = 0.85\,t_d^2$

As s is the same for both dog and car, we can then say that:

$15t_c = 0.85\,t_d^2$

As $t_c = t_d + 1.2$

$15(t_d + 1.2) = 0.85\,t_d^2$

$15t_d + 18 = 0.85\,t_d^2$

Rearranging, we get a quadratic equation for t_d:

$0.85\,t_d^2 - 15t_d - 18 = 0$

With the quadratic solution:

$$t_d = \frac{-(-15) \pm \sqrt{(15)^2 - (4 \times 0.85 \times -18)}}{2 \times 0.85}$$

Solving for t_d we find that the only possible solution is $t_d = 19$ s. So, the dog takes 19 s to catch the car

25. Known: $v_{av} = 60$ km h^{-1} = 16.7 m s^{-1}, $s = 100$ m; to find: t

We can rearrange $v_{av} = \frac{s}{t}$ to get $t = \frac{s}{v_{av}} = \frac{100\ \text{m}}{16.7\ \text{m s}^{-1}} = 6\ \text{s}$

Therefore, the driver will need to take at least 6 s to cover the 100 m to avoid a fine.

26. Known: $s = 10$ m, $t = 0.5$ s; to find: v_{av}

$v_{av} = \frac{s}{t} = \frac{10\ \text{m}}{0.5\ \text{s}} = 20\ \text{m s}^{-1} = 72\ \text{km h}^{-1}$

Therefore, the truck driver will definitely be pulled over for speeding!

27. If the car's average speed is to be 60 km h^{-1} (or 16.7 m s^{-1}), then we can calculate the total time that it will need to take to cover the 50 m:

$t = \frac{s}{v_{av}} = \frac{50\ \text{m}}{16.7\ \text{m s}^{-1}} = 3.0\ \text{s}$

The time taken to cover the first 10 m at 20 m s^{-1} can also be found:

$t = \frac{s}{v_{av}} = \frac{10\ \text{m}}{20\ \text{m s}^{-1}} = 0.5\ \text{s}$

While the driver is reacting, the car continues to travel at 20 m s^{-1} for the next 0.7 s. The distance that the car travels during this time is:

$s = v_{av}t = 20 \times 0.7 = 14$ m

As a result we can see that the car will need to decelerate at such a rate that it will take at least 1.8 s (i.e. 3 s – 0.5 s – 0.7 s) to cover the remaining 26 m (as 50 m – 10 m – 14 m = 26 m).

Thus, as $s = 26$ m, $t = 1.8$ s, $u = 20$ m s^{-1}, we can find a:

$s = ut + \frac{1}{2}at^2$

$26 = (20 \times 1.8) + \frac{1}{2}(a \times 1.8^2)$

$26 = 36 + 1.62a$

$-10 = 1.62a$

$a = - 6.2$ m s^{-2}

Therefore, the car will need to decelerate at a minimum rate of 6.2 m s^{-2} to avoid a ticket.

28. Divide the journey up into 2 parts: A is the first 4 km, which is travelled at 70 km h^{-1} and B is the second 4 km, which is travelled at a speed v, which we are to find.

As $v_{av} = \dfrac{\text{total distance}}{\text{total time}} = \dfrac{s_A + s_B}{t_A + t_B}$,

$$50 = \frac{8 \text{ km}}{t_A + t_B}$$

so $t_A + t_B = \dfrac{8}{50} = 0.16 \text{ h}$

$$t_A = \frac{s_A}{v_A} = \frac{4 \text{ km}}{70 \text{ km h}^{-1}} = 0.06 \text{ h}$$

$$t_B = \frac{s_B}{v_B} = \frac{4 \text{ km}}{v_B}$$

$$0.06 + \frac{4}{v_B} = 0.16$$

$$\frac{4}{v_B} = 0.1$$

$v_B = 40$ km h^{-1}

Therefore, the car will need to cover the last 4 km at a speed of 40 km h^{-1}.